



## Models for estimating volume of *Tectona grandis* at Longuza and Mtibwa forest plantations in Tanzania

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### ABSTRACT

This study aimed to develop and validate allometric equations for estimating total and merchantable volumes of teak (*Tectona grandis* L. f.) trees in two major state-managed forest plantations in Tanzania, Longuza in the Tanga Region and Mtibwa in the Morogoro Region. It addresses the existing gap of the absence of accurate volume models required for the reliable estimation of total teak tree volume, which is critical for effective forest management and planning. A total of 177 destructively sampled trees were used to fit non-linear regression models incorporating diameter at breast height ( $D$ ) and total height ( $H$ ) as predictor variables. Site-specific and general models were developed and evaluated. Models that included both  $D$  and  $H$  outperformed those based solely on  $D$ , providing more unbiased volume estimates across diameter classes and sites. The best-performing models exhibited high explanatory power ( $R^2 \geq 0.96$ ) and low prediction errors ( $PE\% < 3\%$ ). The H-D relationship reveals differences in tree allometry between the two sites, with taller trees on average in Longuza than in Mtibwa. The site-specific H-D and volume equations are recommended where available. In the absence of such models, the generalised equations developed herein provide a reliable alternative across teak plantations in Tanzania.

**Key Words:** Volume equations- teak forest plantations- non-linear model-tree diameter and height relationship- Longuza and Mtibwa.

### INTRODUCTION

The establishment of forest plantations in Tanzania dates back to the 1930s during the German colonial rule in Tanganyika by then. These early initiatives aimed to provide a sustainable source of wood products and to alleviate increasing pressure on natural forests, which were being heavily exploited for timber, fuelwood, and other forest products (Schabel 1990). Over time, forest plantations have become an integral part of the country's forestry sector, playing a vital role in meeting the rising demand for wood-based products while simultaneously contributing to environmental conservation and sustainable land use (Ngaga 2013). In addition to supporting the timber industry, these plantations help to reduce reliance on natural forests and contribute significantly to local livelihoods, particularly in rural communities where forestry-related activities provide employment and income (URT 2022). As of 2022, Tanzania had an estimated 500,000 hectares of forest plantations. Of this total area, approximately 75% is managed by smallholder tree growers and private companies, who have emerged as key players in the commercial forestry sector. These include individuals, community groups, and large-scale private investors who have established plantations mainly in the Southern Highlands. The government, through state-owned forest plantations, owns and manages about 117,000 hectares, which are primarily managed under the Tanzania Forest Services (TFS) Agency and other public institutions (URT 2022).



The forest plantations are planted with various softwood and hardwood tree species, mainly Pines, Teak, and *Eucalyptus* species. *Pinus patula* Schiede ex Schltdl. & Cham occupies about 65% of the country's planted area, followed by *P. caribaea* Morelet (13%), Teak (5%), and *P. tecunumanii* Eguiluz & J. P. Perry. (4%). The remaining 17% is covered by *Eucalyptus* sp, *Cupressus lusitanica* Mill., and *Grevillea robusta* A. Cunn. ex R. Br. occupying about equal share each (URT 2022). Together, these plantations provide a critical source of wood raw materials for construction, furniture, pulp, poles, and energy. Among the key species planted in Tanzania's state forest plantations, teak contributes the most to overall income due to its high commercial value (Kilongo 2021). The state manages two major teak forest plantations in Tanzania at Mtibwa in the Morogoro Region, and Longuza in the Tanga Region. These forest plantations are characterised by differing site conditions, which influence the growth performance of teak trees.

Despite the importance of teak there has been no reliable volume allometric equation. Given this economic importance, there is a critical need to have precise volume allometric equations specifically for teak to support effective forest management, reliable yield estimation, and sustainable harvesting practices. In the context of forest economics and timber trade, tree volume serves as the fundamental pricing unit. Accurate estimation is therefore essential to ensure fair valuation, preventing financial losses for either the buyer or the seller (Avery & Burkhart 2002). Currently, the most widely used volume model for teak in Tanzania developed by Malimbwi *et al.* (1998) using data from Longuza and Mtibwa teak plantations. The equation uses diameter at breast height (D) as the sole predictor variable. In contrast, Zahabu *et al.* (2018) developed biomass and volume equations for teak at Longuza forest plantation that incorporate both D and total tree height (H) as explanatory variables.

Evidences are accumulating that applying a single-volume model that excludes total tree height (H) as a predictor, may lead to biased or inaccurate volume estimates across sites with varying productivity levels (Koch *et al.* 2004, Burkhart *et al.* 2012). This is because H is a key indicator of site quality, and teak trees growing under more favorable ecological conditions tend to be taller than those on poorer sites (Chaturvedi & Khanna 1982, Henry *et al.* 2010a). Therefore, incorporating H alongside D as an explanatory variable is essential for improving the accuracy of volume equations and to ensure they accurately reflect site-specific growth dynamics. Nonetheless, while Zahabu *et al.* (2018)'s model includes both D and H, its applicability is limited to Longuza, as tree height varies significantly between sites, thereby restricting the model's generalizability to be applied elsewhere (Fulton 1999). Efforts to develop teak volume equations have also been undertaken in countries outside Tanzania. These include total and merchantable volume equations for teak in India (Tewari *et al.* 2013), volume estimation models based on stump diameter in teak plantations within the Nimbia Forest Reserve in Nigeria (Shamaki & Akindele 2013), volume equations for teak in the lowlands of Nepal (Koirala *et al.* 2021), and taper equations developed for teak in Western Thailand (Choochuen *et al.* 2021). However, because these models were developed under varying ecological conditions and site-specific growth dynamics, their direct application in the Tanzanian context is unlikely to produce accurate or reliable volume estimates.

Furthermore, the application of tree volume equations that incorporate both H and D requires that H be measured with high accuracy, as errors in H measurement can substantially reduce the precision of volume estimates (Williams and Schreuder 2000). Tree height remains one of the most challenging dendrometric variables to measure due to obstructions from dense crowns, neighboring trees, irregular tree forms, and leaning stems (Larjavaara &



Muller-Landau 2013). These challenges often result in time-consuming data collection and potential measurement errors, particularly in dense forest conditions. Consequently, H-D models are widely used as practical alternatives to predict H from D, providing a reliable and efficient approach for forest inventory and volume estimation (Feldpausch *et al.* 2011, Mehtätalo & Lappi 2020). Despite their usefulness, to the best of the authors' knowledge, no H-D equations specific to teak have been developed for Tanzanian teak plantations.

This study aimed to develop robust models for estimating both total and merchantable volume of teak in two major state-managed forest plantations: Longuza at Tanga region and Mtibwa at Morogoro region. To support the application of these models while minimizing the need for direct H measurements, site-specific H-D equations were also developed. Recognizing the differing site conditions and growth dynamics between the two plantations, the study further assessed variations in volume allometry to determine whether a single generalized model could accurately predict volume across both sites. Additionally, the performance of the newly developed models was compared with existing teak volume equations from Tanzania (Malimbwi *et al.* 1998, Zahabu *et al.* 2018), India (Tewari *et al.* 2013) and Nigeria (Shamaki *et al.* 2011).

## MATERIALS AND METHODS

### Study site description

This study was carried out at the Longuza and Mtibwa forest plantations which are managed by the Tanzania Forest Services Agency (TFS). Mtibwa forest plantation is located in Morogoro Region on the eastern side of the slopes of Nguu Mountains, in

Mvomero District (Figure 1). It lies between latitude 60 - 6010'S and longitude 37040' - 37045' E. The mean annual rainfall in Mtibwa is 1200 mm. Teak was introduced at Mtibwa as a small trial plot of 8 acres in 1936; other experimental plots were established in 1954, 1955 and 1957 at Lusunguru block. A large-scale of teak planting started in 1961 at Mtibwa block. Mtibwa plantation occupies a total area of 16,065.6 ha. For effective management, the area is divided into three blocks, namely, Mtibwa (1,023.6 ha), Lusunguru (2,092 ha) and Pagale (12,950 ha). The main tree species grown is teak covering 95% of the planted area.

Longuza forest plantation is located in the Northeastern part of Tanzania between latitudes 4055' and 5010' S and Longitudes 38040' and 39000' E (Figure 1). The altitude varies from 160 to 560 meters above sea level (m. a. s. l.). Longuza receives 1500 mm of annual mean rainfall. The main tree species grown is teak covering about 96% of the planted area.

### Study design and sampling

Prior to fieldwork, the team consulted forest plantation managers to get acquainted with the forest plantations. Acquired information such as forest compartments register that guided the selection of sample compartments. The selection of compartments was made in such a way that all age and site classes were well represented to ensure coverage of all possible variations. For each compartment, at least two plots (radius = 15 m) were installed, depending on the size of the compartment intended for sample tree selection.

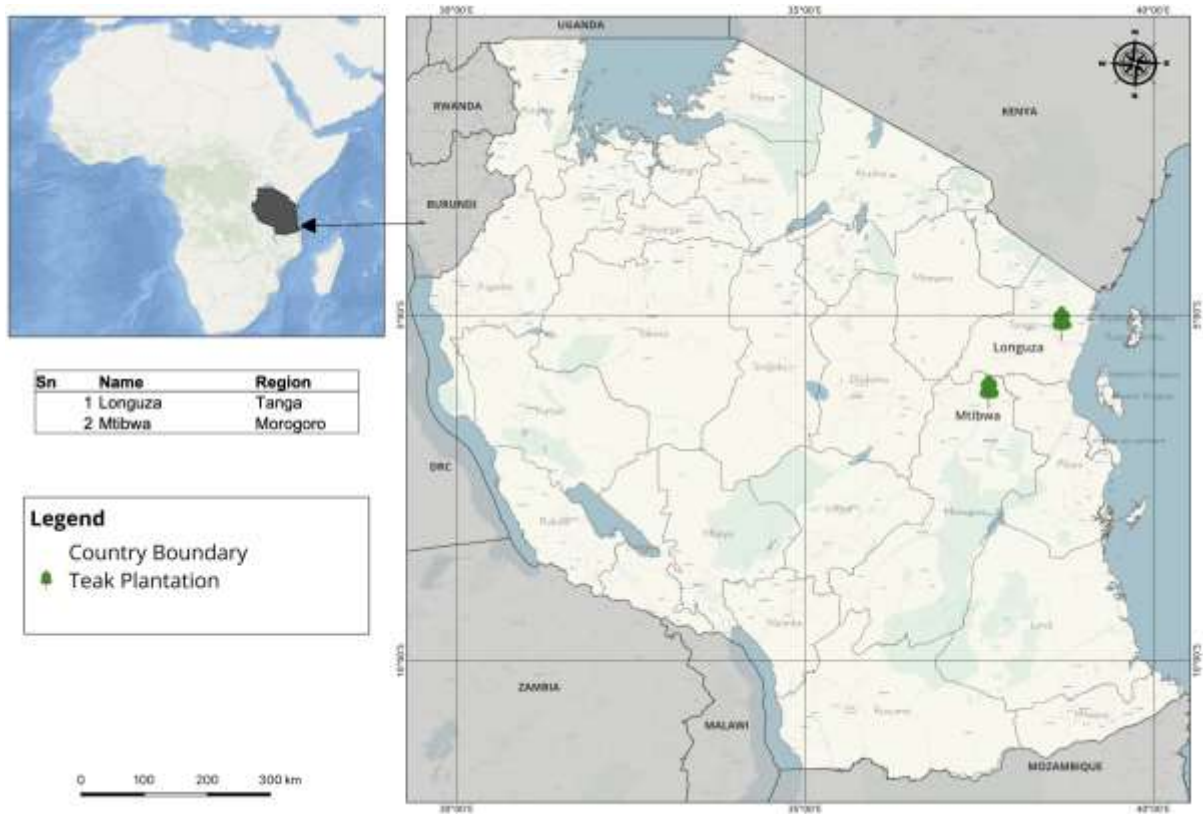


Figure 1. Map showing the location to study sites



### Data Collection

At least two sample trees in each plot were selected, considering their sizes and subsequently felled. Before determining log sections, the sample trees were measured for D and H. Tree sections were determined by marking a log length of at most 4 m. For each section (billets), mid-diameter and length measurements were made and recorded. In addition, the bark was extracted at the mid-section of each log and measured for its thickness. A sample tree was divided into the following compartments: 1) tree section which can produce timber (S, diameter  $\geq 15$  cm); 2) tree section which is merchantable but cannot produce timber (can have a diameter greater than 15 cm but due to other reason such as crookedness cannot produce timber (M)); and 3) a section which is neither merchantable nor cannot produce timber (NM) (includes

branches and cones to a minimum diameter of 3 cm). Table 1 shows the number of sampled trees in each diameter class.

**Table 1. Number of sample trees**

Name of Plantation	Diameter classes	Number of sampled trees
Longuza	10-20	35
	20-30	31
	30-40	9
	40>	16
<b>Total</b>		<b>91</b>
Mtibwa	10-20	39
	20-30	26
	30-40	14
	40>	7
<b>Total</b>		<b>86</b>
<b>Overall</b>		<b>177</b>

### Data Analysis

#### Data Preparation

Preparation of data set for analysis included computation of logs volume using *Huber's* formula. The volume of cones was computed using the volume equation for the cone (e.g., Mauya *et al.* 2014). Total tree volume was obtained by summing up the volume of all log sections. Merchantable volume was obtained as a summation of S and M volume, for trees with age  $\geq 8$  years. In all cases, the over- and under-bark volumes of the tree compartment were computed.

#### Fitting the Volume Equations

The most robust nonlinear models were identified to fit both total volume and merchantable volume models (equation 1-2; Chave *et al.* 2014, Mugasha *et al.* 2016). Site-specific and generalized (combining all the sites) model were fitted. The model forms applied were equation (1), (2) and (3).

$$V_k = a \times D^b \quad (1)$$

$$V_k = a \times (D^2 H)^b \quad (2)$$

$$V_k = a \times D^b \times H^c \quad (3)$$

Nonlinear models were fitted using maximum likelihood (ML) regression techniques which address the problem of heteroscedasticity (Henry *et al.* 2010a, Mauya *et al.* 2014, Mugasha *et al.* 2013). The ML technique fits both model and variance parameters ( $variance = a^2 x^b$ , where  $a$  and  $b$  are parameters to be estimated; and  $x$  is a single or combination of independent variables). The models were fitted using *nlme* package in R (Core Team, 2020). For each fitted equation, Root Mean Square Error (SE) were computed using equation (4), Mean Prediction Error (PE%) using equation (5) and Akaike information criterion (AIC) using equation (6).

$$SE = \sqrt{\frac{\sum_{i=1}^n (V_i - \hat{V}_i)^2}{n}} \quad (4)$$

$$PE\% = 100/n \times \sum_{i=1}^n \left( \frac{V_i - \hat{V}_i}{V_i} \right) \quad (5)$$

$$AIC = 2k - 2\ell \quad (6)$$

where  $V_i$  and  $\hat{V}_i$  are observed and estimated volume of trees for observation  $i$ ;  $n$  is the number of *observations*;  $k$  is the number of all the estimated model parameters, and  $\ell$  is



the value of the log-likelihood. The models with the best AIC values were selected for further evaluation.

### Height-Diameter Equations

Furthermore, we employed a comparable methodology applied to fit volume equations to fit the *H-D* relationship equation. Recognizing the biological characteristics of the *H-D* data exhibiting a sigmoidal pattern, characterized by rapid tree height growth in the early years and subsequent attainment of an asymptote at maturity, the *H-D* relationship was fitted by employing the Weibull model (Weibull 1951, Cooray 2006, Mugasha *et al.* 2019). The Weibull model is presented in equation (7).

$$H = 1.3 + a * (1 - \exp(-b * D^c)) \quad (7)$$

Where *a*, *b*, and *c* are parameters to be estimated, and *exp* is. Exponent. Assessment criteria of the *H-D* models are those presented in equation 4 and 5.

### Model Evaluation

Volume and *H-D* models were evaluated by checking the magnitude of PE%. Evaluation

of site-specific volume model was done by comparing the value of PE% across diameter classes while general volume models were evaluated across sites. Models with larger significant value of PE% were considered to be a biased model. In addition, previously developed volume models inside and outside Tanzania were tested to the current study data by computing the MPE%.

## RESULTS

### Site-specific Total Volume models

Although the PE% for all the models were small and insignificantly different from zero, model 1 in both sites performed relatively poorly with large SE (>0.17) and relatively lower R<sup>2</sup> (<0.94) (Table 2). All coefficients were significantly different from zero (p-value>0.05). Models with a combination of both *H* and *D* had a good fit with R<sup>2</sup> and SE of 0.96 and <0.16, respectively. Due to the smaller value of AIC of model 2 or 3, these models were selected for both sites for evaluation across diameter classes.

**Table 2. Coefficients and performance of site-specific total volume models**

Site	Model	Type	Parameters			Performance			
			a	b	c	R <sup>2</sup>	SE	AIC	PE
Longuza	1	Over bark	0.00011	2.60688		0.94	0.24	-192	3.58
		Under bark	0.000069	2.6934		0.93	0.21	-213	4.36
	2	Over bark	0.00003	1.01039		0.96	0.18	-240	2.4
		Under bark	0.000018	1.045		0.96	0.16	-260	3.06
	3	Over bark	0.000025	1.9341	1.1532	0.96	0.17	-233	2.04
		Under bark	0.0000135	1.9545	1.2711	0.96	0.16	-254	2.49
Mtibwa	1	Over bark	0.000058	2.7756		0.91	0.19	-257	3.34
		Under bark	0.000032	2.8936		0.91	0.17	-285	3.88
	2	Over bark	0.000019	1.0599		0.96	0.12	-326	1.24
		Under bark	0.0000099	1.1074		0.96	0.11	-354	1.56
	3	Over bark	0.0000206	2.1694	0.9837	0.96	0.13	-319	1.5
		Under bark	0.00001	2.247	1.0573	0.96	0.11	-348	1.74



### General Total Volume models

General total volume models' coefficients and performance are presented in Table 3. Model 1 explained 92% of the total variations in volume, while models 2 and 3 explained 96%. Model 1 had a higher SE

compared to models 2 and 3. Based on AIC, model 3 performed better for over bark volume while model 2 performed better for under bark volume. The PE% for all fitted models was not significantly different from zero.

**Table 3. Performance of general total volume models**

Model	Type	Coefficients			Performance			
		<i>a</i>	<i>b</i>	<i>c</i>	R <sup>2</sup>	SE	AIC	PE%
1	Over bark	0.000084	2.6773	-	0.92	0.28	-439.2	3.8
	Under bark	0.000050	2.7753	-	0.92	0.26	-481.4	4.7
2	Over bark	0.000025	1.0305		0.96	0.18	-562	1.95
	Under bark	0.000014	1.0698		0.96	0.17	-604	0.16
3	Over bark	0.000024	2.0344	1.0719	0.96	0.17	-554	1.8
	Under bark	0.0000124	2.0716	1.178	0.96	0.15	-597	2.22

### General Merchantable Volume Models

Total volume model coefficients and performances are presented in

Table . The merchantable volume model had a similar trend in performance to the total volume

models. Similarly, models with both *D* and *H* had larger R<sup>2</sup>, smaller SE and AIC values. Although the PE% differed across the models, they were not significantly different from zero. Based on the AIC, models 2 and 3 were selected for further evaluation.

**Table 4. Coefficients and performance of general merchantable volume models**

Model	Type	Coefficients			Performance			
		<i>a</i>	<i>b</i>	<i>c</i>	R <sup>2</sup>	SE	AIC	PE%
1	Over bark	0.000069	2.6966		0.92	0.27	-216.3	-0.6
	Under bark	0.000037	2.8278		0.91	0.26	-248.8	-0.99
2	Over bark	0.000016	1.0687		0.96	0.17	-318.3	3.24
	Under bark	0.000008	1.1166		0.96	0.16	-355.2	3.93
3	Over bark	0.000014	2.0893	1.1582	0.96	0.16	-311	2.95
	Under bark	0.000006	2.1423	1.2872	0.96	0.15	-349.3	3.32



## Model Evaluation

### Site-Specific Total Volume Models

Across diameter classes, model 1 performed poorly for larger trees. Models 2 and 3 had a well-balanced performance across all diameter classes. Nevertheless, model 3 produced a smaller value of PE across diameter classes compared to model 2 (Table 5).

Therefore, model 3 was finally selected for the estimation of total volume at site level. Performance of merchantable volume models also followed similar trend where model three had a better fit to both sites. The model expressions for volume models for each site are presented in Table 6.

**Table 5. Site-specific models' prediction error across diameter classes**

Site	Class	Mean Prediction Error (PE%)		
		Model 1	Model 2	Model 3
Longuza	10-20'	3.25	2.3	2.04
	20-30'	9.13	5.65	4.59
	30-40'	-12.6	-5.81	-4.82
	40>	13.11	4.73	3.53
Mtibwa	10-20'	7.41	3.07	3.28
	20-30'	-4.68	0.51	0.37
	30-40'	-12.02	-3.71	-3.46
	40>	27.8	7.6	6.66

**Table 6. Site-Specific Total and Merchantable Volume Model expression**

Tree section	Type	Site	Volume model expressions
Total Volume	Over bark	Longuza	$0.000025 \times D^{1.9341} \times H^{1.1532}$
		Mtibwa	$0.0000206 \times D^{2.1694} \times H^{0.9837}$
	Under bark	Longuza	$0.0000135 \times D^{1.9545} \times H^{1.2711}$
		Mtibwa	$0.00001 \times D^{2.247} \times H^{1.0573}$
Merchantable volume	Over bark	Longuza	$0.000017 \times D^{1.9269} \times H^{1.2506}$
		Mtibwa	$0.0000065 \times D^{2.3062} \times H^{1.1785}$
	Under bark	Longuza	$0.0000077 \times D^{1.9498} \times H^{1.4241}$
		Mtibwa	$0.000003 \times D^{2.41053} \times H^{1.2452}$



### Evaluation of General and Previously Developed Volume Models

The findings show that models that include both *D* and *H* perform better in both sites with a *PE* of less than 2.8% compared to a model with only *D* (Table 7). The model with only *D* overestimated total volume by 11.4% and 9.2% for over bark and under bark volume, respectively in Mtibwa. The

previous model (Malimbwi *et al.* 1998) commonly used in these plantations produced higher values of *PE* of 28% and 39% for Longuza and Mtibwa, respectively. Similarly, the ones developed by Zahabu *et al.* (2018) and Shamaki *et al.* (2011) overestimated the total tree volume by over 31% while the *MPE%* for Tewari *et al.* (2013) was below 18%.

**Table 7: Mean Prediction Error (PE%) of general total volume models across sites**

Type	Site	Mean Prediction Error (PE%)						
		Model 1	Model 2	Model 3	Malimbwi <i>et al.</i> (1998)	Shamaki <i>et al.</i> (2011)	Zahabu <i>et al.</i> (2018)	Tewari <i>et al.</i> (2013)
Over bark	Longuza	-1.59	2.54	2.73	28.2	-56.02	-31.41	17.45
	Mtibwa	11.4	2.68	1.68	39.1	-57.67	-59.52	16.41
Under bark	Longuza	-1.3	2.72	2.79	-	-	-	-
	Mtibwa	9.2	1.13	0.75	-	-	-	-

It was important to evaluate the performance of the models over a wide range of diameter classes. The evaluation results show that models with both *D* and *H* perform better overall in all diameter classes compared to models with only *D* (Table 8). The performance of the two models with both *D* and *H* (models 1 & 2) was equally matched, except that model 2 slightly overestimated the volume of trees in the smallest and largest diameter classes. The previously used model (Malimbwi *et al.* 1998) performed poorly, particularly on the small diameter classes with *PE* reaching up to 61% and 49% at Longuza and Mtibwa,

respectively. Shamaki *et al.* (2011) and Zahabu *et al.* (2018) had poor performance overall with higher overestimation and underestimation of larger diameter trees, respectively. The *MPE%* for model developed by Tewari *et al.* (2013) was below 24% in all diameter classes.

It should also be noted that based on the value of *PE%* generated across diameter classes, site specific models were slightly better than general models (Table 5; Table 8).



**Table 8: Mean Prediction Error (PE%) of merchantable volume models across diameter classes**

Site	Diameter classes (m)	n	Mean Prediction Error (%)						
			Model 1	Model 2	Model 3	Previous developed volume equations			
						Malimbwi <i>et al.</i> (1998)	Shamaki <i>et al.</i> (2011)	Zahabu <i>et al.</i> (2018)	Tewari <i>et al.</i> (2013)
Longuza	10-20	35	-9.7	-1.0	-0.6	49.6	-25.0	-45.1	-15.5
	20-30	31	-4.7	5.1	5.0	28.6	-36.6	-30.6	-17.9
	30-40	10	-16.3	-3.6	-2.8	-6.0	-70.0	24.6	-14.2
	40>	15	1.6	8.9	8.6	-0.5	-85.7	145.1	-23.1
Mtibwa	10-20	39	10.4	8.1	6.9	61.2	-17.6	-62.0	-22.6
	20-30	26	-6.2	0.3	0.0	28.8	-38.5	-56.4	-13.0
	30-40	14	-5.2	-7.1	-8.1	9.7	-60.8	3.1	-7.4
	40>	7	16.3	1.0	-1.6	13.2	-74.5	117.6	-15.3

These findings confirm the importance of  $H$  in volume estimation. The  $H$  is the common variable used to depict the site classes. It is well known that Longuza is the better site for teak compared to Mtibwa, i.e., relatively taller trees and higher volume in Longuza compared to Mtibwa (Figure 2). Since the volume variation in these sites is mainly driven by  $H$  (site quality),  $H$  in the fitted models was able to pick up and explain volume variations between teak growing at Longuza and Mtibwa. It is no wonder that models with  $H$  included were the best. Based on this evaluation, model 3 (equation 6: total volume over bark; and equation 7: total volume under bark) is recommended to be used to estimate the total volume of teak in both Longuza and Mtibwa.

$$V_{Under\ bark} = 0.0000124 \times D^{2.0716} \times H^{1.178} \quad (6)$$

$$V_{Over\ bark} = 0.000024 \times D^{2.034467} \times H^{1.0719} \quad (7)$$

Where  $V_{over\ bark}$  is total volume over bark;  
 $V_{under\ bark}$  is total volume under bark.

### Evaluation of General Merchantable Volume Models

Results for evaluation of merchantable volume models are presented in Table 9. The performance followed a similar trend to total volume models. Models 2 and 3 performed better compared to Model 1 in explaining the merchantable volume of teak.

### Height-Diameter Relationship

The parameter estimates and performance metrics of the H-D models are presented in Table 10. Site-specific H-D models outperformed the general model, as indicated by higher  $R^2$  values, lower standard errors (SE) and mean prediction errors (MPE%). The asymptotic height for Longuza (50.2 m) was notably higher than that of Mtibwa (27.3 m). The general model yielded prediction errors of 4.1% for Mtibwa and 7.5% for Longuza (results not shown).



**Table 9: Evaluation of Merchantable Volume Models across sites**

Type	Site	Mean Prediction Error (PE%)		
		Model 1	Model 2	Model 3
Over bark	Longuza	-6.93	2.80	3.00
	Mtibwa	5.92	3.69	2.89
Under bark	Longuza	-7.57	3.33	3.66
	Mtobwa	5.78	4.54	2.98

**Table 10. Parameter estimates for site-specific and general H-D models**

Site	n	Equation. Expression	SE	R2	MPE%
Longuza	91	$H = 1.3 + 50.1824 * (1 - \exp(-0.0379 * D^{0.8537}))$	2.37	0.87	0.6
Mtibwa	86	$H = 1.3 + 27.37515 * (1 - \exp(-0.0175 * D^{1.3675}))$	2.13	0.85	1.2
General	177	$H = 1.3 + 37.5825 * (1 - \exp(-0.0322 * D^{1.0170}))$	2.64	0.82	4.2

## DISCUSSION

This study utilized a dataset of 177 destructively sampled teak trees, deemed sufficient to capture variations in volume allometry resulting from differing growth conditions across sites. Trees were systematically selected from multiple compartments at Mtibwa and Longuza, covering a range of site qualities and age classes. The description of data reveals differences in tree volume allometry as well as the *H-D* relationship between the two sites, with relatively taller trees and higher volume in Longuza compared to Mtibwa, as shown in Figure 2. Therefore, a stratified sampling approach ensured robust statistical representation and adequately accounted for variability in site productivity and tree development factors known to influence key growth parameters such as *D*, *H*, and form (Henry *et al.* 2010b, Chave *et al.* 2014). Incorporating such diversity enhances the reliability and generalizability of the developed volume equations, making them

suitable for effective site-specific forest management planning (Picard *et al.* 2012).

## Volume Model Performance

For all the fitted sectional volume models developed in this study, the results demonstrate that incorporating *H* in addition to *D* improves model performance. Specifically, the inclusion of *H* led to a noticeable reduction in the SE of the volume models, increased the R<sup>2</sup>, and improved the ability to predict volume (lower MPE%). This finding is consistent with previous studies in forest plantations, such as Mugasha (2025), who reported enhanced accuracy in teak volume estimation at rotation age when *H* was included alongside *D*. Similarly, research conducted in natural forests supports the same conclusion; for instance, Mauya *et al.* (2014) and Mugasha *et al.* (2013) found that models incorporating both *D* and *H* provided better



predictive power for tree volume and biomass in Tanzanian miombo woodlands.

### Height - Diameter Relationships

The findings demonstrate that the site-specific  $H$ - $D$  models outperform the general model, suggesting that  $D$  alone does not fully capture the variation in  $H$  between the two sites. This underscores the importance of accounting for site-specific factors and supports the use of site-specific  $H$ - $D$  models for more accurate height estimation in volume calculations. The findings further show that the average maximum  $H$  that can be attained at Longuza is relatively higher, i.e., 50 m compared to Mtibwa, i.e., 37 m (Figure 2). This is likely due to the fact that, while trees may exhibit the same growth pattern in  $D$ , their  $H$  may differ due to several factors, including management practices (e.g., initial planting spacing and thinning) and site-specific growth

conditions such as soil fertility, rainfall, and elevation. In this case, however, site growth conditions, such as soil depth and nutrients, as well as precipitation (Mtibwa: 1200 mm versus Longuza: 1500 mm), are likely to be responsible for the observed pattern (Yanzhong *et al.* 2011). On the other hand, at local scale, in a densely planted stands often encourage trees to grow taller as they compete for light, while wider spacing can lead to shorter, broader trees (Zeide 1993; Sharma & Parton, 2007). Similarly, site quality has a strong influence on tree architecture, where trees growing on more productive sites tend to be taller than those on less favorable sites, even when their diameters are similar (Feldpausch *et al.* 2011, Banin *et al.* 2012). These variations underscore the importance of incorporating both  $H$  and  $D$  into volume allometric models to enhance the accuracy of volume estimation, particularly in heterogeneous forest conditions.

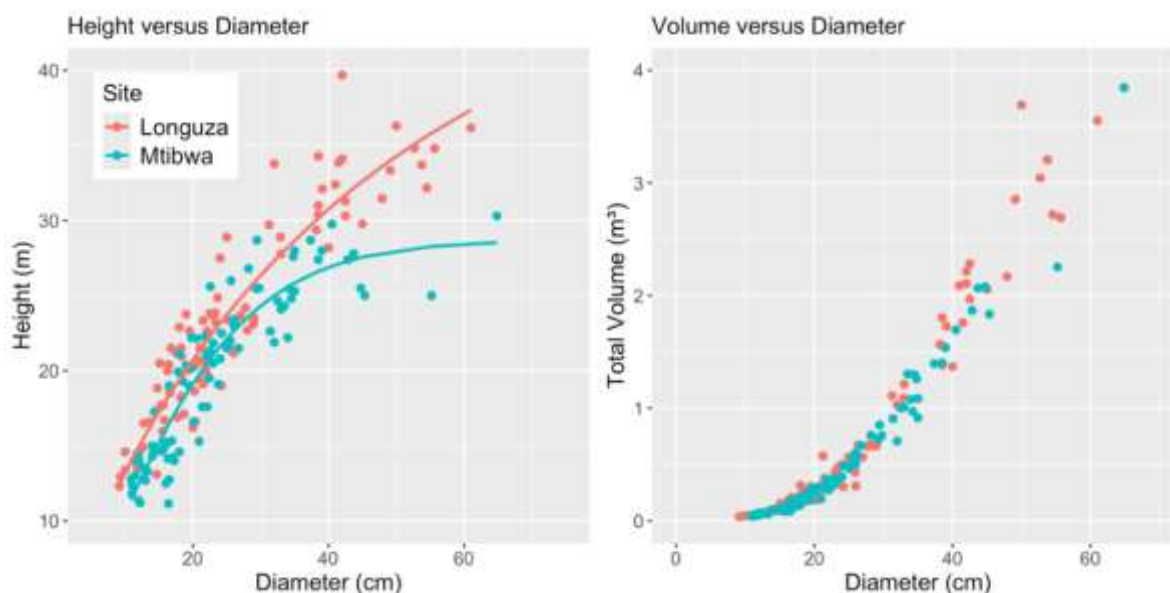


Figure 2. Allometry of teak grown at Longuza and Mtibwa



## Evaluation of Site-Specific, General, and Previously Developed Tree Volume Models

The model evaluation findings indicate that the selected site-specific volume models incorporating both  $D$  and  $H$  provided a good fit and yielded unbiased MPE% across all diameter classes. In contrast, models that used only  $D$  as a predictor consistently produced biased MPE% values, particularly for larger trees. This highlights the critical role of  $H$  in volume estimation, as height variations are more pronounced in larger or older trees, and failing to account for this variable can lead to systematic prediction errors (Banin *et al.* 2012, Mugasha *et al.* 2019).

The evaluation of the selected general models revealed that those incorporating both  $D$  and  $H$  yielded unbiased estimates across all sites, whereas models using  $D$  alone underestimated volume in Mtibwa. This suggests that the volume general model, which incorporates both  $D$  and  $H$ , can accurately estimate volume in all study sites, demonstrating the added value of  $H$  in accounting for between-site variation (Burkhart and Tomé 2012). The poor performance of the volume equation that excludes  $H$  has been demonstrated by the evaluation findings of the previously developed volume model (Malimbwi *et al.* 1998), which produced a biased volume estimate with MPE% > 20%. The other evaluated volume models exhibited a minimum bias of 17%. Several factors may have contributed to the high MPE% observed in our dataset. These include variations in planting materials (Wondimneh *et al.* 2024) and differences in management practices, such as initial spacing and thinning regimes (Aquino *et al.* 2021). An exception is the model developed

by Zahabu *et al.* (2018), which utilised data from Longuza. Given this, better performance would have been expected when applied to the current dataset. However, the model's limited accuracy may be attributed to the small sample size ( $n = 44$ ) used in its development and lack of deliberate stratification (Chave *et al.* 2004), particularly considering the substantial height variability among trees within the same site, as illustrated in Figure 2.

## CONCLUSION

The study successfully developed and validated tree volume equations for teak at two major plantation sites in Tanzania using non-linear regression models. Models that incorporated both  $D$  and  $H$  as predictor variables consistently outperformed those based solely on  $D$ , demonstrating higher accuracy and stronger statistical performance. The derived equations exhibited high  $R^2$  and low PE%, confirming their reliability for use in forest inventory data, planning, and sustainable management of teak plantations. The  $H$ - $D$  relationship reveals differences in tree allometry between the two sites, with taller trees on average in Longuza than in Mtibwa. Site-specific  $H$ - $D$  and volume equations are recommended where available, as they provide greater precision for both  $H$  and volume estimation, respectively. In the absence of site-specific models, the general equations developed in this study offer a robust alternative for broader application.

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