

Effect of $\mathbf{E} \times \mathbf{B}$ drifts in convective zone

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Abstract. The $\mathbf{E} \times \mathbf{B}$ drift allows plasma to move through the magnetic field lines and may contribute to various motions inside the Sun (e.g. to explain the adverse gradient of differential rotation in the equatorial zone), at its surface and in the corona. Here we treat an example: using a given azimuthal angular frequency $\omega(r, \theta)$, rather arbitrary, and the corresponding exact solution for \mathbf{B} , we obtain \mathbf{E} and the drift velocity. The latter is comparable with the original velocity, but has components in all directions

1. Introduction

Magnetohydrodynamics (MHD), especially non-ideal MHD, is a quite good theory in a large domain of applicability. However, it is not always fully exploited. In fact MHD is based on the subset of the full system of equations:

$$\begin{aligned}\partial_t \mathbf{H} &= \text{curl}(\mathbf{v} \times \mathbf{H}) - \text{curl}(\eta \text{curl} \mathbf{H}), \\ \text{div} \mathbf{H} &= \mathbf{0},\end{aligned}$$

which contains only \mathbf{H} from the electromagnetic quantities and only \mathbf{v} from the hydrodynamical ones. ($\mathbf{B} = \mu \mathbf{H}$, with μ taken as constant.) However, this does not mean that there are no currents flowing. Indeed:

$$\mathbf{j} = \text{curl} \mathbf{H}.$$

The current affects the motion as it appears in the Lorentz force. Moreover, an electric field \mathbf{E} appears through another Maxwell equation:

$$\text{curl} \mathbf{E} = \mu \partial_t \mathbf{H}.$$

The $\mathbf{E} \times \mathbf{B}$ drift yields a velocity

$$\mathbf{v}_d = \mathbf{E} \times \mathbf{B} / |\mathbf{B}|^2,$$

which applies equally to electrons and ions and allows the plasma to move easily across the field, contrary to the common belief that plasma is strictly tied to the magnetic field lines.

2. Exact solution for \mathbf{H} from $\omega(r, \theta)$

Suppose that the velocity (due to the inertia of the matter) is purely azimuthal

$$v = r\omega(r, \theta) \sin \theta,$$

(r, θ, ϕ : spherical co-ordinates) with angular frequency ω . We have shown (Callebaut & Callebaut 1991) that the integration of the equations leads e.g. to (omitting periodic terms)

$$\begin{aligned} H_r &= (\partial_\theta U)/r^2 \sin \theta, \\ H_\theta &= -(\partial_r U)/r \sin \theta, \\ H_\phi &= (t/r)\partial(\omega, U)/\partial(r, \theta), \end{aligned}$$

in which $U(r, \theta)$ is an arbitrary function determined by the initial field.

3. Illustration of $\mathbf{E} \times \mathbf{B}$ drift

The Maxwell equation for \mathbf{E} yields $E_\phi = 0$ as we use $\partial_\phi = 0$, and

$$\partial_r(rE_\theta) - \partial_\theta E_r = -\mu\partial(\omega, U)/\partial(r, \theta).$$

Hence

$$\mathbf{E} \times \mathbf{B} = \mu(E_\theta H_\phi, -E_r H_\phi, E_r H_\theta - E_\theta H_r).$$

Clearly this may lead to drifts in any direction, depending on the choice of U and the resulting \mathbf{E} . To fix the ideas we consider a very simple example: $\omega(r)$, $U(\theta)$ and $E_r = 0$. Then $E_\theta = -\mu\omega\partial_\theta U/r$ where we neglected the "constant" of integration. For the drift velocity we obtain

$$\mathbf{v}_d = [-t\omega(\partial_r\omega)r^2 \sin^2 \theta, 0, -r\omega \sin \theta]/(1 + r^2 t^2 \sin^2 \theta (\partial_r\omega)^2).$$

The azimuthal velocity corresponds roughly to the speed of rotation (2 km/s at the equator), but, after say a month, it fades away. The radial velocity increases up to the same speed and fades then away. Note that the choice of the initial field has no influence here because of its dependence on one co-ordinate only.

4. Conclusions

There is still a lot of arbitrariness in the choice of the initial field, in ω , in the integrating "constant". Moreover, the new velocity pattern requires an iteration. Although the illustration has still a limited meaning we may say that the resulting drifts are of the order of magnitude of the generating velocities. It is clear that the $\mathbf{E} \times \mathbf{B}$ drift should be taken into account in the motions in (cf. the adverse gradient of angular frequency in the equatorial zone) and outside the Sun, (e.g. explosive phenomena, (Vezelovsky 2004)). To some extent this is an attempt to go beyond the customary MHD, still on the basis of MHD.

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